| Common Misconceptions | Intervention Strategies |
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| The student may use long strings of equations which incorrectly reinforces the idea that the equals sign means "the answer is coming." | - Have students use a balance scale or an EquaBeam balance to practice making both sides have the same value. Use precise language to emphasize that the equals sign means the numbers/expressions on either side of the equals sign have the "same value." |
| Students may think that they simply add zeros to the end of a number when multiplying by multiples of ten. | - Practice looking for patterns as they multiply by ten and multiples of ten. Each time you move left starting from the ones place, to a new digit, the value of that digit is tens times greater (multiply by ten) than what it represents in the place to its right. Students can explore this relationship that is modeled by these equations: $1 \times 10=10 ; 10 \times 10=100 ; 100 \times 10=$ $1000 ; 1,000 \times 10=10,000 ; 10,000 \times 10=100,000 ; 100,000 \times 10=1,000$, 000. <br> - Provide base-ten pieces and a place value chart to model how the value of digits changes by powers of ten as the digits shift. (See the Place Value Chart in Unit 3 Lesson 5) <br> - Provide students with sentence frames to encourage the use of precise mathematical language. For example: <br> - When the number $\qquad$ is multiplied by 10, each digit shifts one place to the left. <br> - When the number $\qquad$ is multiplied by 100, each digit shifts two places to the left. When the digit $\qquad$ shifts from the (tens) place to the (hundreds) place, the digit becomes 10 times as much as it was. |
| Some students might use the word "and" when saying a whole number. For example, they may read the number 28,956 as twenty-eight thousand and nine hundred and fifty-six. | - Reinforce that the word "and" is only used when expressing part of a number (fraction or decimal form). For example, the number 28,956 should be read as twenty-eight nine hundred fifty-six. <br> - Discuss the period names that students will be using when reading numbers in the place value system (units, thousands, millions) and how a |



\section*{The student may regroup ten ones to create a ten but forget to adjust the value. For example, he may solve $268+175$ like this: \\ | 268 | 268 | He regrouped ten ones to make a new ten but |
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| +175 | +175 | forgot to adjust the number of tens from 3 tens |
|  | 3133 | to 4 tens. |

This error indicates that the student does not understand the value that a digit represents when regrouping or renaming in addition and subtraction problems. This misconception often occurs when students are taught algorithms for adding and subtracting multi-digit numbers before they understand the concept of place value and what the digits actually represent.

- Provide place value charts and base-ten pieces to model conceptually the regrouping of ones for tens and tens for hundreds. Allow time for students to represent with drawings and symbolic notation of the addition problems after concrete modeling.
- First, you add the digits in the ones place which is 13 . Since 13 is $10+3$, you put the three in the ones place and the ten in the tens place.
- Next you add all of the tens: $10+60+70$ which is 140 since $140=$ $100+40$, you put the four tens in the tens place and the hundred with the hundreds.
- Finally you add the hundreds: $100+200+100$ which is 400 .
- So all together you have a sum of 443 .
- Have the student use the Expanded Form to model the addition problem: $268+175$.
$268=200+60+8=$
$+175=+100+70+5=$
$300+130+13=300+(100+30)+(10+3)=(300+100)+(30+10)+3=$
$400+40+3=443$
- Use games in which students bundle groups of objects into 100s and 10s and then record the number of bundles of 100 s and 10 s, the number of single objects remaining, and the number it represents.
- Have students represent a 2-digit or 3-digit number in as many possible ways as they can using $100 \mathrm{~s}, 10$ s and 1 s . This can be done by modeling, pictures, or symbols. For example, the number 48 can be represented as 4 tens and 8 ones, 3 tens and 18 ones, 2 tens and 28 ones, 1 ten and 38 ones, 0 tens and 48 ones. The number 253 can be represented as 25 tens and 3 ones, 20 tens and 3 ones, 2 hundreds and 53 ones, etc.
- Have the student use the Expanded Form to model the subtraction problem: 374-139.
$374=300+70+4=300+(60+10)+4=300+60+(10+4)=300+60+14$

| 374 $\mathbf{3 7 4}$ He ungrouped one ten but forgot to adjust <br> the tens from 7 tens to 6 tens. He subtracted <br> $\underline{-139}$ $\underline{-139}$ (tens minus 3 tens instead of 6 tens minus <br> 3 tens. | $\underline{-139}=\underline{-100+30+9}=\underline{-100+30+9}=\underline{-100+30+9}=\frac{-100+30+9}{200+30+5}=235$ |
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| A student may have difficulty using rounding as an estimation strategy to determine the reasonableness of answers. | - Emphasize that rounding should be based on number sense and place value instead of using a memorized rule. <br> - Focus first on rounding 4-digit numbers to the nearest thousand first because it is less challenging than rounding 4-digit numbers to the nearest hundred. <br> - Show students a set of 4-digit numbers such as, 2380, 1000, 3160, 4000, 1250, 2000, 3000, and 3990. Ask students to decide which of the numbers are easier to work with and why. They can turn and talk with a partner. Have students share their thinking with the whole group. <br> - Use the following set of 4-digit numbers: 3000, 4000, 3500, 3290, and 3870. Make a number line with a piece of string. Place a card with 3000 on the left end and a card with 4000 on the right end. Ask students to discuss with a partner where the 3500 card should be placed on the number line. After reaching consensus, place the 3500 card at the midpoint of the number line. Have students decide where the other cards ( 3290 and 3870 ) should be placed on the number line. Ask: "Why did 3290 round to 3000 and 3870 round to 4000 ?" Have students think of other numbers that would round to 3000 or 4000 . |
| The student may think that subtraction is commutative, for example $5-3=3-5$. When a student solves a subtraction problem such as $73-25$, he sees that 3 ones are less than 5 ones and decides to calculate 5-3=2 rather than ungroup a ten to create more ones. If the student makes this error, he will find the difference of 73-25 to be 52 instead of 48 . Since $5+4$ and $4+5$ both equal 9 , it is understandable how students may assume that 10-25 and 25-10 are equivalent expressions. However, unlike its | - Have the student use concrete objects (that represent hundreds, tens, and ones: base ten blocks, dollar bills- $\$ 100, \$ 10, \$ 1$ ) to model the subtraction problem such as $732-265$. When you start with 7 hundreds 3 tens and 2 ones, you need to create access to more ones by trading a hundred for ten tens and a ten for ten ones which results in 6 hundreds 13 tens and 12 ones. Then 5 ones can be subtracted from 12 ones and 6 tens can be subtracted from 13 tens. The resulting difference is 467 . |


| inverse operation, subtraction is NOT commutative. These misconceptions can be overcome by ensuring that students have a solid mathematical understanding of subtraction rather than relying on shortcuts or memorized algorithms. | - The student can use a place value mat and base-ten blocks to model solving subtraction problems. <br> - Using the Expanded Form makes the regrouping easier to see. Have students solve problems in which ungrouping/regrouping occurs. In third grade, students learned to "fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction." In fourth grade, students are learning to fluently add and subtract multi-digit whole numbers using the standard algorithm with place value understanding. $\begin{aligned} & 213-105= 213 \\ & \underline{-105} \end{aligned}$ <br> Because the ones digit in the total (3) is less than the ones digit in the subtrahend (5), a student may make the error of changing the order of the ones digits to 5-3=2. <br> Solving with the Expanded Form: $\begin{aligned} 213 & =200+10+3 \\ -105 & =\frac{-100+0+5}{100+10+2}=112 \end{aligned}$ |
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| A student may only solve one part of a two-step word problem and think he is finished. | - The magnitude of the numbers in the story problem can be lessened to make it easier for students to focus on the structure of the problem. <br> - Have students read, reread, and discuss the word problem with a partner or in a small group. <br> - They should think about the following questions: <br> - What do the numbers in the problem represent? <br> - What information do you need to figure out? <br> - Why do you think you need to (add, subtract, multiply or divide) to solve the problem? <br> - Can you solve this problem in one or two steps? <br> - How do you know? |

The student may have difficulty solving elapsed time problems that cross over the hour because students must use regrouping skills to convert between hours and minutes ( 1 hour = 60 minutes).

- Use an open number line to help students visualize the beginning time, duration, and end time in a linear way. Provide interactive modeling by counting forward or back on the number line by hours and minutes.
- Explore adding and subtracting time at a rate of 1 hour equals 60 minutes which is not based on powers of 10 .

Resources:
Faulkner, V. N. (2013). Common Core.
https://www.engageny.org/sites/default/files/downloadable-resources/2014/Dec/why the common core changes math instruction.pdf
Karp, K. S., Bush, S. B., \& Dougherty, B. J. (2014). 13 Rules That Expire. Teaching Children Mathematics, 21(1), 18-25. http://ps186.org/wp-content/uploads/13-Rules-that-Expire.pdf

Rutherford, Kitty and Schulz, Denise: NC Department of Public Instruction . "Common Mathematical Misconceptions"
Van de Walle, John. Teaching Student-Centered Mathematics (Volume 2 - Grades 3-5).
"Misconceptions and Errors," Mathematics Navigator.
"Fourth Grade Instructional Framework" - North Carolina Collaborative for Mathematics Learning (nc2ml.org)
Tools4NCTeachers website

